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VARIABLE STARS[1]

THE speaker before such a gathering as this, in this eventful year, faces a dilemma in his choice of a subject. The topic which is foremost in all our minds is, beyond a doubt, the share which our comrades in science have had in carrying to a triumphant close the great work of the war—and an account of this would in some respects be the most suitable subject for a vice-president's address. But most of this work can not be described yet, if at all, for reasons of military secrecy; and it is still too early, in any event, to collect and correlate the records of the work of men who are still in the service, especially when almost the whole of the narrator's time has been spent in attempting, in a very humble way, to aid in the universal effort.

I have therefore chosen the opposite horn of the dilemma, and propose to speak to you to-day upon a topic of pure science—removed perhaps as far as anything could be from the theater of war, trusting to whatever intrinsic interest the subject may possess to atone for the lack of timely interest, and the defects incident to hurried preparation.

Variable Stars have been the objects of human wonder since the appearance of the Nova of Hipparchus led to the preparation of the first catalogue of the positions and magnitudes of the stars. The period of scientific observation of these changes may be dated from Tycho Brahe's observations of the Nova of 1572 and Fabritius' discovery of the periodic variation of *Mira Ceti* in 1596.

For two and a half centuries after this date the number of known variables remained so small that they could almost have been

[1] Address of the vice-president and retiring chairman of Section A—Astronomy—of the American Association for the Advancement of Science, Baltimore, December 27, 1918.

counted on one's fingers. Since then the more assiduous observation of modern times has raised the number into the hundreds, and the application of mechanical methods of search (that is, of photography) has swelled our list into the thousands, with prospects of further increase.

It is now possible to classify almost all variables into fairly definite—often very definite—natural groups. In each of these groups more or less numerous empirical regularities of behavior, or "laws" have been detected; and in some instances the number of these relations is large, and the accuracy with which they represent the characteristics of individual stars is surprising. It has even been possible to use these relations to deduce information regarding the distribution of the stars in space, for example, which has revolutionized our previously existing ideas. Yet the humiliating admission must still be made that, in spite of these advances, we know extremely little of the real causes of stellar variation. A satisfactory theory exists in the case of but one group—and this is based on the fundamental assumption that the stars of this class are not really variable at all, but owe their apparent changes to the geometrical accident of eclipse! Of the causes, mechanism and physical relations of the intrinsic variability of the stars we are still in dense ignorance. Could we solve the riddle, there is good reason to hope that the key to some of the fundamental problems of astrophysics would be found.

In the study of variable stars, therefore, we have a series of problems which are at once laborious, difficult, fascinating and of great promise, and a summary survey of the field may afford very appropriate material for this address—considering first the methods of observation, and then the known facts, together with such theoretical conclusions as may be drawn from them.

From the observational side, the study of variable stars affords excellent examples of the advantages of "scientific management" and of cooperation. Until about a generation ago, the discovery of variables was made by accident, in the course of other work—such as the making of meridian catalogues or star charts, or the search for asteroids,—and the rate of discovery was naturally small. The great superiority of photography for this purpose was first effectively realized by Pickering and his assistants at Harvard. By superposition of a positive of a given star-field upon a negative of the same field taken at some other date any variables which may be present can be picked out at once. More than a thousand variables have been discovered at Harvard in this way—a number far exceeding the total which all the astronomers of the world had found by visual means in the preceding three centuries. The process is so easy that Miss Leavitt and her associates when working up a new region, never trouble to identify the previously known variables, but simply rediscover them along with the rest. From the percentage of known variables which are missed (usually a small one) it is possible to estimate how many unknown ones have been passed over and await future discovery.

Similar photographic studies of globular clusters led to Bailey's important discovery of the presence of variables in them—the importance of which is only now being fully realized. Mention should also be made of the successful work of Max Wolf and others with the blink microscope.

Another fruitful method of discovery is by means of spectrum photographs with the objective prism. Certain types of spectra with bright lines are practically certain to belong to variable stars. Mrs. Fleming and Miss Cannon have thus discovered some two hundred variables of long period and about half of the known galactic novæ.

When however a variable star has been discovered, the observer's work has just begun. Its changes must be followed and their laws determined. In many cases the variations are found to be exceedingly regular, both in period and amplitude, so that a very precise mean light curve may be obtained. Such

stars may profitably be followed by photometric methods of precision, as in the visual observations of Wendell and of Dugan and his students, or by means of "physical photometers," such as the selenium cell, or the photo-electric apparatus, the last of which especially, in the hands of Stebbins and of Guthnick, has proved to be the most precise of all ways of measuring starlight.

When such high precision is not necessary, good results may be secured by Argelander's method of direct estimate of the brightness in comparison with neighboring stars of known magnitude—either directly with the telescope, as Luizet and Roberts have done with notable success, or upon photographs, as by Miss Leavitt at Harvard.

Here the great Harvard Library of photographs is a rich mine, worked to but a small fraction of its capacity for lack of miners. Estimates of the photographic brightness of a few stars discussed at Princeton, show that the probable error of a determination of brightness from a single plate is about ± 0.06 magnitudes. Material for thoroughly reliable light curves of all regular variables which are brighter than the eleventh magnitude at minimum already exists in this great collection, and it is urgently desirable that more workers should be provided to make it available.

Still higher accuracy can be obtained upon plates taken specially for photometric purposes, as has been shown by Plummer and his associates.

The majority of variable stars, however, are far from exactly regular; and, when successive maxima may differ in an unpredictable fashion by a whole magnitude, there is clearly little advantage in observing to hundredths.

There are hundreds of such stars, most of them observable with small telescopes over a good part at least of their range—and to follow them all would tax the resources of the regular observatories severely. In this field amateurs have been able to make what is probably their most noteworthy contribution in the whole range of present day scientific activity. The American Association of Variable Star Observers, and its older colleague, the Variable Star Section of the British Astronomical Association, have organized this amateur activity in a highly successful fashion, and observations of these previously rather neglected objects are pouring in at the rate of many thousands a year—affording material which will be of inestimable value in the future.

The observations of variable stars show that almost all of them fall into some one of five natural classes—to adopt the very convenient division devised at Harvard. In the order there used these are: I., *Temporary Stars,* or Novæ; II., *Variables of long period;* III., *Irregular variables;* IV., *Short period variables,* or *Cepheids,* including cluster variables; and V., *Eclipsing variables,* including the Agol and Beta Lyræ types.

Variables of the last two classes are strictly periodic, and notably regular in their changes; those of the first three classes are not. But before discussing the separate classes in detail, we may well consider some of the general properties of variable stars of all kinds.

First, it is noteworthy that, speaking at large, there is as wide a range in the spectral types of variable stars as in those of the stars as a whole. Every one of the principal spectral types, and almost every subdivision of these types, is represented among variables. It appears safe to conclude from this that reliability, *per se,* is not confined to any one particular stage of stellar evolution.

There is however a decided difference in this respect between the various classes of variables. Eclipsing variables, though mostly of classes A and B, are found as far down the sequence as class K. Cepheid variables are known through the whole range from B to M—that is, in all the principal spectral classes.

Irregular variables, on the other hand, are almost all of classes M and N—and therefore among the reddest of the stars.

Long period variables also belong, without exception, to these spectral classes, and the great majority of them to the subclass Md, showing bright hydrogen lines, at least when

near maximum. Spectra of this type are practically certain evidence of variability.

Finally the novæ are distinguished by very peculiar and characteristic spectra, which undergo equally characteristic variations as their light fades. A very few other stars which show spectra of similar character are also variable (η Carinæ and the nuclei of two variable nebulæ).

Our first generalization may therefore be stated in the form:

Regular periodic variability has apparently little connection with the evolutionary stage of a star's history, while variation of a roughly periodic or non-periodic type appears to be intimately associated with particular stages of development.

This is after all very much what might be expected, for regular variation keeping time accurately suggests a process regulated by gravitation or rotation, and hence not necessarily connected with any stage of evolution, while other forms of variation may well arise from the physical state of a star, and appear only at definite evolutionary stages.

A second general property of variable stars is that, with insignificant exceptions[2] they are objects of great luminosity, far exceeding the sun. The eclipsing variables, with a few exceptions, average more than fifty times as bright as the sun. The irregular variables, and the long-period variables at maximum, appear to be comparable in brightness with other naked eye stars of class M, and hence about a hundred times as bright as the sun. The Cepheids are among the brightest stars of which we know, ranging from 100 up to perhaps 10,000 times the sun's brightness. As for the Novæ, we know as yet but little, but that little indicates that, at maximum, they may be even brighter than the Cepheids.

In general, it appears certain that almost all variable stars are what Hertzsprung so felicitiously calls "giants." According to the theory which the speaker has had a share in advancing this would mean that variability, while not confined to any one stage of a star's evolution, is a characteristic of its early life—not the flickering of the dying flame of age, but the exuberance of extravagant youth.

[2] A few eclipsing variables, and the stars in the nebula of Orion recently discussed by Shapley.

Hertzsprung's suggestion that the very faint dwarf red stars, which, all agree, represent the last observable stages of stellar history, should be investigated for possible variability, deserves however more attention than it has so far apparently received.

With these preliminaries, let us turn to a rapid survey of the individual classes of variables. Here it will be convenient to reverse the Harvard order, and begin with the *eclipsing variables.*

Typical variation of this sort is immediately recognizable, since it consists in regular interruptions of otherwise almost constant brightness. This behavior suggested to Goodricke, more than a century and a quarter ago, that the obscurations of Algol were due to partial eclipses by a huge dark planet, revolving around the luminous star in the period of variation. As this hypothesis considerably antedated the discovery that true binary systems existed among the visual double stars, it was of striking originality, and it may fairly be claimed that the history of binary stars begins with Algol. Unfortunately, the very boldness of the hypothesis led to its neglect for a full century, until Pickering revived it, and Vogel's spectroscopic study of Algol gave it striking confirmation; and it is only within the last ten years that the study of eclipsing binaries has really come into its own as a branch of double-star astronomy coordinated with that of visual or spectroscopic systems.

The eclipse theory of the variation of stars of this type now stands on about as firm foundations as anything in modern astronomy, being confirmed (1) by the precise representation of numerous well observed light curves (the irregularities whose presence was previously suspected disappearing with improved methods of observation); (2) by the fact that every eclipsing variable which has so far been studied spectrographically proves to be a spectroscopic binary in which the

period and phase of the orbital motion, and the relative brightness of the secondary, are in agreement with the photometric data, and (3) by the success of the reverse process of photometric investigation of promising spectroscopic binaries, which have in many cases revealed variation of the eclipsing type, though of small amplitude, but with the theoretical period and phase. The fact that some spectroscopic binaries have proved not to be variable is a further confirmation of the theory (the orbital inclination being such that eclipses fail to happen).

The development by the speaker of simple methods for computing the orbital elements of these systems has extended our list so that, at present, owing mainly to Shapley's industry, orbits are available for about 100 eclipsing pairs—a sufficient number to allow of drawing conclusions by statistical methods. These stars show a strong concentration toward the galactic equator, and the majority of them have spectra of classes A and B, though there are a number of class F and a few of classes G and K. Their periods are usually less than ten days, though two are known with periods of about six and nine months, and several others with periods about a month.

When a more careful study is made of their variations it is found that in every case in which decisive observations have been made, the eclipsing companion is not a dark body, but a self-luminous star. The maximum difference in brightness between the components of a pair is about four magnitudes, which would not be considered very great in the case of a visual binary. The secondary minimum, due to the eclipse of the companion by the principal star, is almost always of observable depth, and has been found whenever properly looked for.

In most of the cases which have so far been studied, the faint companion is of greater diameter than the brighter primary; but it is very doubtful whether this represents the general rule among close binary systems, for pairs in which the fainter star is the smaller can at best show but a small range of variation, and few of them are likely to be discovered, especially as observers give the preference to the stars of large range of variation.

It appears however to be an invariable rule that the faint companion is always much *redder* than the primary—which is clear evidence that its faintness is due to lower temperature. This is confirmed by Miss Cannon's direct observation, that in U Cephei, a typical system of this class, the spectrum of the brighter star is of class A, while that of the much fainter one which totally eclipses it at minimum is of class K. Two stars separated by a space equal only to their diameters, and doubtless of common origin and equal age, may therefore differ as widely in spectral type as do Sirius and Arcturus.

These systems offer the only direct method at present available for finding the relation between the color index and surface brightness of a star. The investigation has so far been complicated by uncertainties regarding the color equation of both the visual and photographic observations of such faint objects. When this difficulty has been surmounted, as it soon should be, there is a good prospect of being able to determine the surface brightness of a star from its color index—which would clear the way for determining the linear diameters of all stars of known parallax, and the angular diameters of all the stars in the heavens.

Of no less importance is the information which eclipsing binaries alone can give about the densities of the stars. The numerous stars of this sort, and of spectra A and B, are remarkably similar in density, averaging about one sixth the density of the sun. The fewer pairs of spectra F, G and K show a much greater range of density—some being denser than any of the stars of the first type, while others are of very low density, in a few cases less than 1/100,000 of the sun's. There is also strong evidence that the large faint red components of the systems mentioned above are less massive, and hence much less dense, than the small, bright, primaries of "earlier" spectral type. The bearing of these facts on stellar evolution is obvious.

By combining this knowledge of the densi-

ties of these stars with estimates of their masses and surface brightness (which can be made with a fair degree of confidence), it is possible to estimate their parallaxes, and study their distribution in space. The results of Shapley and the speaker show that these stars are distributed through a region nearly symmetrical about the galactic plane. The great majority of them lie within two or three hundred parsecs of this plane, but they extend along the plane in all directions to a distance of at least 1,500 parsecs, with no sign that our observation has reached the limit.

The statement that the brightness of an eclipsing variable is constant in the intervals between eclipses is only approximately true. In many instances—notably in Beta Lyræ—there are definite maxima half way between the eclipses, indicating that the stars are elongated into ellipsoids by their mutual tidal influence. The amount of this ellipticity decreases rapidly for pairs in which the components are more widely separated, following very closely the law predicted theoretically by Sir George Darwin.

Again, there is often a "radiation effect" showing that the companion keeps the same face always toward the primary, and that this face, being heated by the later, is brighter than the opposite side of the secondary (as was observed by Dugan in RT Persei, and by Stebbins in Algol). In a few cases, when the observations are very precise, it has been possible to show, from the form of the light curve, that the disk of the principal star is not uniformly luminous, but is brighter at the center than at the limb, like that of the sun.

The theory of eclipsing variables is therefore now in a satisfactory state: but there is much which still remains to do. Dozens of stars are still to be observed, and the light curves of many more require more accurate determination. The important task of comparing the visual and photographic light-curves of the same stars is only just begun; and there is a wide field for the physical photometer among the recently discovered systems of small range. Theoretically, many problems of interest await solution, especially the explanation of the small but indubitable, and perplexingly complicated, changes in period which occur in most of the systems which have long been observed. If these changes can be definitely referred in some cases to motion of the line of apsides of the orbit, it should be possible to obtain information about the degree to which the central density of the stars in question exceeds their mean density—a matter about which we now know nothing. There are also curious discrepancies between the times of minima, as observed with light of different colors, which are very puzzling. Enough is already known, however, to furnish a direct observational proof that the velocities of light of different colors in empty space can not differ by more than a very few meters per second.

Spectrographic observation of all accessible eclipsing variables for radial velocity is urgently to be desired, for this affords the only known way of adding to the few cases in which we can determine the actual diameters of the stars. Such matters as the precise determination of spectral type, especially of the faint companions during total eclipses, and the study of absolute magnitudes by spectroscopic means also deserve attention.

The Cepheid variables, which come next in order in our survey, present one of the most attractive and difficult problems of present day astronomy. There are few other fields in which we know so many facts, and can explain so few. The mass of information is so extensive that it must be rather summarily presented. Observation shows that:

1. The Cepheids show a regular variation, For each star the period is constant, and the light curve repeats itself with remarkable regularity. (The small deviations from this rule suspected by Shapley and others, though doubtless real, are insignificant in comparison with the irregularities that occur among long-period variables).

2. The periods range from three hours to more than a hundred days, but show two well-marked maxima of frequency at about twelve

hours (the cluster variables) and seven days (the Cepheids proper). The two groups are almost separated, there being very few stars with periods about two days.

3. The range of variation is always small, very rarely over 1.5 magnitudes. Photometry of precision is revealing many cases of Cepheid variation of very small range, the most notable example being Polaris.

4. The variation is continuous, and the rise to maximum is usually much more rapid than the fall to minimum, while the maximum is usually more sharply accentuated than the minimum. There are a few exceptions to the last rule, and also a few stars in which the rise and fall of brightness are about equally rapid, but none in which the rise is decidedly slower than the fall. The light curve is usually smooth and flowing, and secondary maxima or minima are rare, though they appear to be authenticated in a few cases. The general form of the light curve (which greatly resembles that of the velocity curve of a spectroscopic binary) is characteristic, and makes it easy to identify variation of this type.

5. The photographic range of variation is always greater than the visual, the curves being similar in shape, but the photographic amplitude about 50 per cent. greater. These stars are therefore much redder at minimum than at maximum.

6. The spectra of the stars of period less than a day are almost all of class A, (β Cephei, with a period of 5^h is of class B), those of three or four days' period are of class F; of 10 days' period of class G; while those of longest period are of class K or M.

7. The spectral class of these stars varies with the light, being "earlier" at maximum and "later" at minimum. As measured by the hydrogen lines, the range of variation is nearly a whole spectral class on the Harvard scale: as measured by the metallic lines it is much less.

8. All these stars show variation of radial velocity, with the period of the light changes. The epoch of minimum radial velocity (most rapid approach) coincides closely with that of maximum light, and that of maximum radial velocity with minimum light—no exception having yet been found among some twenty stars.

9. The range in radial velocity is nearly proportional to that in visual magnitude, at the rate of about 47 km. for one magnitude. Hence the velocity curve can be roughly drawn when the light curve is known. The correspondence of the two, however, is not precise.

10. Most of the brighter of these stars belong to Miss Maury's *ac* class, *i. e.*, have spectral lines sharper than the average. According to Adams' recent criteria, the spectra of these stars indicate very high luminosity.

11. The Cepheids proper (of period greater than two days) show a strong concentration toward the galactic equator, and have very small proper motions which nevertheless show conspicuous evidence of the drift due to the sun's motion in space. Their peculiar velocities are small, about 10 km./sec.

12. Those variables of the "cluster type" which are found in the sky at large behave very differently, showing a small galactic concentration. In the few cases so far studied their proper motions are considerable, (though they are faint stars) and their peculiar velocities very large.

13. A large number of Cepheid variables occur in the Small Magellanic Cloud, and a very definite relation exists between their periods and photographic magnitudes, the stars of longer period being the brighter. As the stars in the Magellanic Cloud must all be at very similar distances, this indicates that the absolute magnitude of a Cepheid is a function of the period.

14. Among the numerous short period Cepheids in the globular clusters those in anyone cluster are of almost exactly the same median brightness—differing from cluster to cluster. It is therefore very probable that all "cluster variables" of period less than a day, are of the same mean absolute magnitude. The few Cepheids of long period which occur in globular clusters are considerably brighter.

15. From the parallactic motions of the Cepheid variables which occur in Boss's Pre-

liminary General Catalogue it appears that these stars, which have a mean period of about seven days, have a mean absolute magnitude of —2.3 on Kapteyn's scale. Roughly speaking, they average at maximum about 1,000 times as bright as the sun and at maximum about 500 times the sun's brightness.

By combining these data, Shapley, in a remarkable series of papers, has derived an empirical curve which gives the absolute magnitude of any Cepheid whose period is known. The cluster variables are of median absolute magnitude —0.2 and average about 100 times as bright as the sun. The Cepheids of longest periods, forty days and over, appear to be fully ten thousand times as bright as the sun. With the aid of these data he has shown that the Cepheids proper lie in a region only a few hundred parsecs thick, which extends along the galactic plane for several thousand parsecs in all directions, while the isolated variables of the cluster type spread out into space far on each side of this region. The same data have been fundamental in his determination of the distances and distribution in space of the globular clusters—which has revolutionized our conceptions of the extent of the universe, and of the relation of the naked eye stars to it, but would lead us too far from the theme of the present hour.

Our empirical knowledge of Cepheid variables is therefore both extensive and accurate, and has already led to astrophysical conclusions of far-reaching importance. But our understanding of the nature of the physical process which lies behind the variation of these stars lags far behind in the rear.

Some things seem fairly clear. The concomitant variations in brightness, color and spectrum indicate very strongly that the proximate cause of all three is a variation in the surface temperature of the star. The enormous magnitude and great rapidity of the changes (a cluster variable may increase its brightness by fifty times the sun's whole light in two hours, and lose all this again in six hours) make it probable that the changes in temperature must arise from some process by which heat energy is transformed periodically back and forth into some form of potential energy—the loss by radiation during one period being relatively insignificant. Finally, the regularity of the process indicates that the regulative force behind it is gravitational—the potential energy taking some from such as an expansion of the mass against gravity—and that the physical process involved is some form of gravitational oscillation of "pulsation" of the mass, probably involving the compressibility of the material as an essential factor.

There are various alternative hypotheses, some of them attractive, for example, that which assumes that we have to do with a rotating body, hotter and brighter on one side than the other. This explains the general character of the changes in light and spectrum, and their regularity; but a detailed analytical study of the problem shows that it is impossible for the rotation of a convex body, brighter in some parts than others, but with each portion of invariable brightness, to give rise to a light curve in which the rise is as rapid, compared with the fall, as in the case of many typical Cepheids, and all the cluster variables. Hence it appears certain that actual changes in the temperature and brightness of the surface of these stars must take place during each period. The variations in radial velocity are also very hard to explain on the rotation theory, which would give the wrong phase.

Another attractive hypothesis, which meets the difficulty about the radial velocities, assumes that the face of the star which precedes in the orbital motion is brighter than the hinder side, as would be very probable if the motion took place in a resisting medium. But this idea appears less alluring when it is realized that these stars, which are very similar to the sun in spectrum and color, and presumably in surface brightness, must in that case have radii of the order of 20 million kilometers, while the average radius of one of the spectroscopic orbits is less than two million kilometers. Hence the stars themselves are in all probability much larger than their orbits

(just as the radius of the earth exceeds that of the orbit which the earth's center describes about the center of gravity of the earth and moon). If, as is very probable, their periods of rotation and revolution are the same, the actual motion of one of these stars would closely resemble a rotation about an axis passing nearly, but not quite, through its center, and there would be no "leading side" in the sense assumed by the theory.

Moreover, the rotation of so large a body would give rise to an equatorial velocity so large as to make all the lines in the spectrum wide and diffuse; whereas they are actually narrower than in most stars.

It therefore appears improbable that these bodies are really spectroscopic *binaries*, and more likely that the line displacements arise in some other manner than from orbital motion.

A further argument against both these theories is that, if the direction of the rotation or orbital motion in any system should be reversed, the resulting variation would show a slow rise to maximum and a rapid fall—which has never been observed. If the variation arose from orbital motion the same effect would be produced by observing the star from any point on our line of sight, but behind it. That variations of this sort should be of equal geometrical probability to the others, and yet never be observed among a hundred cases, is altogether beyond reason.

Though these theories appear therefore to be unsatisfactory, it must be frankly admitted that the pulsation hypothesis is not yet in a position to explain positively the form of the light-curve or the apparent variations in radial velocity. So far, all that can be said for it is that it does not seem to fall foul of any obviously fatal difficulties, and that it appears likely to be fairly flexible.

A detailed mathematical study of the possible modes of vibration of a compressible, gravitating, radiating and possibly rotating mass of gas may lead to the solution of the problem. In spite of the evident difficulty of such a discusion it is to be hoped that it will soon be attempted—perhaps by the new and brilliant school of English mathematicians of which Eddington and Jeans are the leaders.

In the present state of our knowledge, the following admittedly speculative considerations may be of interest.

From Shapley's careful work it appears that, among the Cepheids proper, the absolute magnitude and color index are practically linear functions of the logarithm of the period. For a star of color index 0.75 (similar to the sun's) the period is 7 days, and the median absolute magnitude —2.3, corresponds to a luminosity 700 times that of the sun. It seems fairly safe to assume that the surface brightness of such a star is equal to that of the sun, so that its diameter may be estimated as 26 times the sun's.

Cepheids of longer periods are brighter and redder, the absolute magnitude decreasing (numerically) by $1^m.0$, and the color index increasing by $0^m.4$ if the period is doubled. This increase of color index indicates almost certainly a decrease of surface brightness. From the existing data, it sems probable that the change in surface brightness, expressed in stellar magnitudes is fully three times that in the color index. To obtain a total luminosity one magnitude greater, with a surface brightness 1.2 magnitudes less, the diameter must be increased 2.7 times. Hence it appears probable that a Cepheid of 14 days period is of about 70 times the sun's diameter, while one of 40 days' period (about the longest usually met with, of about twice the diameter of the earth's orbit—very large, it is true, but undoubtedly still of stellar and not of nebular dimensions.

A typical cluster variable 100 times as bright as the sun, and of color index 0.15, should be of about four times the sun's surface brightness, and four and a half times its diameter.

If now we adopt the pulsation hypothesis, it follows that, as in the case of all gravitational oscillations, the period will be inversely proportional to the square root of the density.

Hence doubling the period corresponds to a four-fold diminution of the density. But we

have just seen that it also gives with an increase in diameter by a factor of 2.7 or of volume by a factor of about 21. Hence it follows that Cepheids of any given period have about five times the mass of those of half the period. This conclusion (which holds good only for the Cepheids proper, and not at all for the cluster variables) depends to some degree upon the assumed relation betwen color index and surface brightness; but it is hard to see how any juggling with the data can escape from the conclusion that Cepheid variation sets in, if at all, at different stages of evolution among stars of different masses—the most massive stars reaching the critical condition, (whatever it may be) at the lowest density.

Among the cluster variables, which are very similar to one another in color and brightness, and hence probably in diameter, both density and mass must be greatest for the stars of shortest period. This reversal of the relationship may be connected with the fact noted by Shapley, that the absolute magnitude of these stars mark an inferior limit of luminosity, below which Cepheid variation appears not to occur.

To attempt a numerical estimate of the densities and masses of these stars is precarious, as we do not know the exact nature of the mode of pulsation. But the assumption seems plausible that for a star of given density, the period of the Cepheid pulsation is not likely to be very different from that of the fundamental oscillation of a gaseous mass of the same density. According to Emden, this period would be two hours for a mass of the density of the sun. This leads to the rough estimate that a Cepheid of 7 days period is of about 1/7,000 of the sun's density. With the diameter previously estimated this would mean a mass 2.7 times that of the sun. A Cepheid of $3\frac{1}{2}$ days' period would have about half the sun's mass, and one of 40 days' period about 150 times the sun's mass. A cluster variable of 12 hours period would be of 1/36 the density, and $2\frac{1}{2}$ times the mass of the sun.

These numerical values are extremely uncertain but it is of interest to note that they appear to be of quite a reasonable order of magnitude. The masses calculated for the Cepheids of moderate period, which are brilliant giant stars, are probably rather low—which suggests that the Cepheids of long period are really stars of exceptionally great mass, as would follow, on Eddington's recent theory, from their exceptionally great luminosity. Eddington has very recently called attention to another important deduction that may be drawn from the study of variables of this type. The period of Delta Cephei is shortening by about one second in twenty years. On the pulsation hypothesis, this would indicate a gradual increase of density, but at so slow a rate that it would take the star three million years to double in density, and probably ten million to pass from class G (its present type) to class F.

If studies of the secular variation in period of other Cepheids confirm this date of change, there will be direct evidence that the rate of stellar evolution is exceedingly slow and that the time scale of cosmical processes is of very great length.

But what time remains must be devoted to the other classes of variable stars. The *irregular variables* need detain us but for a moment, for, beyond the fact that their variations are usually of small amplitude, their spectra practically all of classes M or N, and their luminosities probably comparable with those of other giant stars of these types, we know practically nothing about them.

The variables of long period form a very definite natural class, with periods ranging from about 80 to 800 days, but exhibiting a nearly normal frequency distribution about a mean value of 300 days. The range of variation is much greater than for any other class of variables except the Novæ, averaging about four magnitudes, and sometimes reaching seven or eight—a change of a thousand-fold in light.

They are far from being regular time-keepers. For almost all the stars that have been carefully followed, the times of maxima deviate from any uniform period by much more

than the errors of observation. Attempts to represent these deviations by means of empirical formulæ have so far failed to meet the test of prediction, and it begins to look as if they were, in the strict sense of the word, irregular—though of the fundamental periodicity lying behind them there can be no doubt.

The maximum brightness, and the details of the form of the light curve, also differ very considerably at different times for the same star. The mean of a number of periods, however, usually gives a fairly smooth light-curve of roughly sinusoidal form, but with the rise usually steeper than the fall. Harmonic analyses of these curves have been made by Turner and Phillips, showing that the first harmonic largely predominates though the second and third are usually quite sensible. Phillips has shown that nearly all the stars so far investigated fall into one or other of two groups, marked by certain definite relations between the phases of the second and third harmonics. In other words the deviations of the light curve from a simple sinusoid tend to follow one or other of two definite patterns. This discovery will doubtless prove of theoretical importance, but no attempt to explain it can yet be made.

The above remarks apply to the curves obtained by plotting the stellar magnitude against the time. If the actual light-emission should be used instead, the curves would have high, steep-pointed maxima and very flat minima, and their representation by a Fourier series would demand a host of harmonics. The simple character of the curves giving the magnitude as a function of the time suggests, as Plummer has pointed out, that the periodic process involved may be something, such as variation in the thickness of an absorbing layer, which would directly affect the logarithm of the escaping light. But such a layer would have to absorb over 90 per cent. of the energy passing through it for months at a time without becoming greatly overheated, which seems, hard to believe.

The most notable spectroscopic features of these stars are the presence of heavy flutings of titanium oxide, indicating very low surface temperatures, and of bright hydrogen lines—which are especially strong near maximum; and, on the negative side, a conspicuous absence of changes in radial velocity.

The rather scanty data at present available indicate that the peculiar velocities of these stars are very high, and their proper motions moderate or small in amount. This would indicate considerable distance and luminosity, and it seems clear that, at maximum, these are fairly bright giant stars, at the least. Further data, especially regarding proper motions, are much to be desired, and should be obtainable in many cases.

Very little is known of the real causes of variation of this type. It is certainly not due to orbital motion, and the prevalent irregularities suggest strongly that we have here to do, not with a gravitational or rotational phenomenon, but with a physical process—something resembling in nature the eruptions of a geyser, when an accumulation of internal energy piles up to the point where it obtains relief through some overlying resistance, giving rise to roughly periodic outbursts of varying intensity. The relatively great length of the period falls in well with this hypothesis. There is much about the phenomena which suggests the variation in solar activity of which the sunspots are the most prominent symptoms. This is pretty certainly due to some such accumulation of internal strain and has the same irregularly periodic character, though a much longer period.

What the actual nature of the process is which can change the brightness of a star by several hundred fold remains, however, for the future to determine.

Mention should be made in passing of two curious types of variation, each represented by but few stars, but very definite, which are classified at Harvard as subdivisions of class II. The typical star of one of these groups. R Coronæ Borealis is usually of about the sixth magnitude, but at irregular intervals drops rapidly to the eighth, tenth, or even the thirteenth magnitude, recovering again after intervals which may be months or years in duration. Two or three other stars behave in

the same way. The other group, typified by U Geminorum and SS Cygni, are normally very faint, but at irregular intervals (of two or three months for the last named star) increase rapidly by some four magnitudes, to fade away again after a few days. It is a curiously suggestive fact that the stars of each of these singular classes are very similar to one another in spectrum, while the spectra of each class as a whole are quite unlike one another or anything else in the heavens. Here is indeed a riddle for the future to solve.

Finally, we come to the temporary stars—those most spectacular of all celestial objects. To discuss them fully would require another address comparable in length to this. The merest outline must suffice. Before the outburst, several of them are known to have existed as faint stars, often slightly variable. Without warning and within a very few days at most, their light increases at least a thousand fold—sometimes fully a hundred thousand times. The happy chance by which the recent great Nova in Aquila was caught midway in the rise indicates that its whole ascent occupied about two days. The maximum brightness is sometimes very great.—Tycho's star of 1572 equalled Venus—but a rapid decline sets in almost at once, followed by irregular oscillations with a general downward tendency, merging into a slower but steady decline, till within a decade or so the star has lost eight or ten magnitudes and returned nearly to its original brightness.

The spectroscopic changes are meanwhile of the most extraordinary character. The three stars which have been caught on the rise showed dark line spectra, roughly resembling familiar types, but no two alike, and with the lines greatly displaced, as if by a huge velocity of approach. As the star goes "over the top" its spectrum explodes, so to speak, in a few hours into a flamboyant affair of bright and dark lines, enormously widened and displaced, and undergoing continual changes. Lines of hydrogen, helium, and enhanced metallic lines have been recognized. Besides the bright hydrogen lines in Nova Aquilæ there were at times two sets of sharp dark lines—apparently due to hydrogen, but displaced by amounts corresponding to velocities of approach of about 1,800 and 2,600 km./sec. Complicated changes occur as the light fades, the most important being the appearance of the characteristic nebular lines, which at some stages are the most conspicuous feature of the spectrum, and remain visible for a long time. After some years, however, they begin to fade, and the last state so far recognized is spectroscopically identical with the Wolf-Rayet stars.

Novæ show a very strong galactic condensation. Nothing is known of their proper motions, or (for obvious reasons) of their peculiar velocities; but direct measures of parallax indicate that the distances of some of the brighter ones are of the order of at least 100 parsecs. They must therefore be exceedingly bright objects at maximum; but how bright we do not know.

These objects bear very remarkable relations to nebulæ. They appear to be related spectroscopically to the gaseous nebulæ. The unique moving nebula near Nova Persei was admirably explained by Kapteyn as due to the illumination of a sheet of diffuse matter, nearly at rest, by the outgoing light of the great outburst—a hypothesis confirmed by Slipher's recent discovery that two variable nebulæ appear to shine by reflected light from their nuclei, which show spectra very similar to novæ.

Most remarkable of all is the recent discovery that novæ appear in the spiral nebulæ so fast that it would take intensive observations to catch them all.

It is obvious that in these temporary stars we are in the presence of *catastrophes,* which in magnitude utterly transcend all other known physical phenomena. And these catastrophes are not of rare occurrence, but happen every few years, or oftener, in the galaxy, and apparently every few weeks in the Andromeda nebula. Two possibilities suggest themselves at once—a collision or an internal explosion. Collisions between two stars are quite out of the question—owing to the frequency and

short duration of the phenomena. The hypothesis of a collision between a star and a nebula meets these two fundamental objections, and appears capable of accounting qualitatively for many or most of the phenomena, as was shown some years ago by Seeliger. But the spectroscopic data, and especially the dark-line spectrum on the rise, remain difficult to explain. A collision between a star and a relatively small dark body—recently postulated by W. H. Pickering—is also worthy of consideration, but presents difficulties of its own.

After what we now know and believe regarding the stores of energy which are locked up in the nuclei of atoms, the hypothesis of an explosive release of some such form of energy within a star can not be neglected. The chief difficulty about it seems to be that we might expect an even greater catastrophe than appears to occur—but this theory will probably prove to be increasingly flexible as our knowledge advances. At present, however, the collision theory appears to the speaker the most promising. The great frequency of novæ in the spiral nebulæ—where we might expect collision to occur, if anywhere in the universe—seems to be favorable to this view.

In concluding this hasty and imperfect survey of a wide field, two things stand out prominently—first, the importance of a study, which was once neglected and even rather despised, in the attack upon some of the most fundamental problems of astrophysics, and, second, the urgent need of extensive and active researches, observational, statistical and theoretical, to advance toward solutions of some of the many unsolved problems which still remain before us.

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ON this side of the Atlantic there have been few zoologists who have devoted their lives to the study of ancient fishes—which for the rest concerns not a few of the greatest problems of the vertebrates. Of investigators who have passed away we recall the distinguished names of Agassiz the elder, Cope, Newberry and Leidy, and to this goodly fellowship we must now add the name of Charles Rochester Eastman, whose services have contributed widely and intensively to a knowledge of fossil fishes. To this work he gave his time devotedly for a quarter of a century, publishing over a hundred papers, among them a number of monographs which rank among the most scholarly and accurate in their field.

Eastman graduated from Harvard in 1891, studied at Johns Hopkins, thereafter in the University of Munich, where he took his doctorate in 1894; he worked with Professor Karl von Zittel, whose laboratory then attracted a number of young American paleontologists. Here, as Eastman's interests already centered in fossil fishes, he was given the only material for research which the German university had at hand—a mass of detached teeth of a Chalk Measures shark—not attractive material, to say the least, but the young investigator attacked it with energy and soon gathered the data for a successful thesis. He was next given a post at Harvard, where in the Museum of Comparative Zoology, under the mantle of Louis Agassiz, he reviewed the collections of early fishes and found much material for publication. He now became interested in the Devonian fossils of the Agassiz collection, which he found shed light upon the rich finds from the Middle West, then being described by Dr. Newberry. Eastman's imagination was especially touched by the range and character of "placoderms" as the dominant group of Devonian times, and like many another worker, he set himself to solve the puzzles of their lines of evolution and of their kinship to modern fishes. Hence he sought actively for more extensive and better preserved material upon which to base his findings. The best collecting ground for these American forms was in Ohio, and throughout this region Eastman soon learned to know the fossil hunters and their collections. His studies upon these forms thereupon spread over wider fields, and became well-nigh encyclopædic; he brought the entire Devonian

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